

# Planar front instabilities during directional solidification of hcp: Zn–Cd dilute alloys

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## Abstract

The cellular and dendritic growth of Zn–Cd dilute alloys is directly influenced by crystalline orientation, the primary orientation being the  $\langle 10\bar{1}0 \rangle$  and eventually  $\langle 0001 \rangle$ . In general, this information is absent during the description of planar to cellular transition. However, it has been experimentally shown that the critical velocity for the stability limit, and the associated wavelength of the first instabilities, are different depending on the growth direction. In this work the stability of the accelerated planar fronts of Zn–0.01%Cd samples, obtained using a Bridgman-like device, and the behavior against morphological stability theory has been compared.

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## 1. Introduction

Microstructure formation during directional solidification of dilute alloys is a very important process, both from an academic point of view and for technological applications. A recent review was given by Boettinger et al. [1]. The morphological stability theory [2–4] defines the conditions under which an initially planar solid–liquid interface could become unstable and therefore form different microstructures.

In the case of dendritic structures, the presence of an anisotropic surface energy allows the structure to define unique operative conditions, fixating the relation between the tip radius and primary spacing. Also, this anisotropy has a fundamental role in the formation of the cellular growth.

Numerous works have been carried out in this way, mainly in face-centered cubic (fcc) crystallography dilute alloys. From them, it is possible to define different morphologies [5–7] as a function of solidification parameters:  $G_L$ , thermal gradient at liquid;  $V$ , interface advance velocity;  $C_0$ , nominal composition of the alloy. For alloys that are more crystallographically complex, the morphology obtained during directional solidification may show a different behavior depending on the growth direction. Besides, a precise description of microstructural evolution does not exist for hexagonal close-packed (hcp) dilute Zn–Cd alloys; only a few descriptions appear in the literature [5,8–10].

For hcp crystals, the preferential growth direction is  $\langle 10\bar{1}0 \rangle$  as Fig. 1 shows schematically. When the heat extraction does not coincide with this direction, there could be a change in the critical conditions as an effect of that crystalline anisotropy. During planar to cellular transition this results in a change in the critical characteristic values [9–11].

The interfacial energy can be written in three dimensions as

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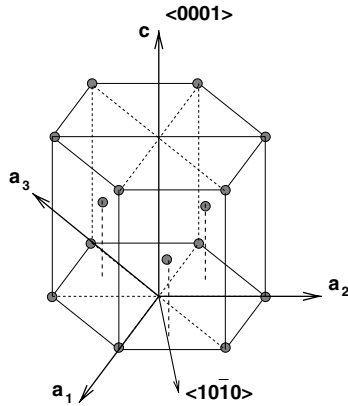


Fig. 1. Crystalline structure of an hcp crystal, where the preferential growth directions  $\langle 0001 \rangle$  and  $\langle 10\bar{1}0 \rangle$  are shown.

$$\Gamma(\hat{\mathbf{n}}) = \Gamma_0[1 + \epsilon_0(n_1^4 + n_2^4 + n_3^4)] \quad (1)$$

where  $\Gamma_0$  is the isotropic part of superficial tension,  $\epsilon_0$  indicates the anisotropy degree, which for metals is of the  $\approx 0.065$  order [12], and  $\hat{\mathbf{n}}$  is a unity vector locally normal to the interface.

If we choose, as usual, the  $\langle 0001 \rangle$  as the azimuthal direction [13] and then measure the relative directions from here, the angular form of the superficial tension  $\Gamma(\theta, \phi)$  can be written as a function of the spherical harmonics

$$\Gamma(\theta, \phi) = \Gamma_0[1 + \epsilon_0 Y_{l,m}] \quad (2)$$

Fig. 2 schematically shows the relation  $\Gamma(\theta, \phi)/\Gamma_0$  when it is calculated for  $Y_{33}$ , assuming a three-fold symmetry in the polar  $x$ - $y$  plane [14]. As the figure shows, the relation  $\Gamma(\theta, \phi)/\Gamma_0$  can be greater or less than unity in different zones. In particular, a decreasing value of  $\Gamma(\theta, \phi)$  appears in the  $\langle 0001 \rangle$  direction whereas there is an increasing value

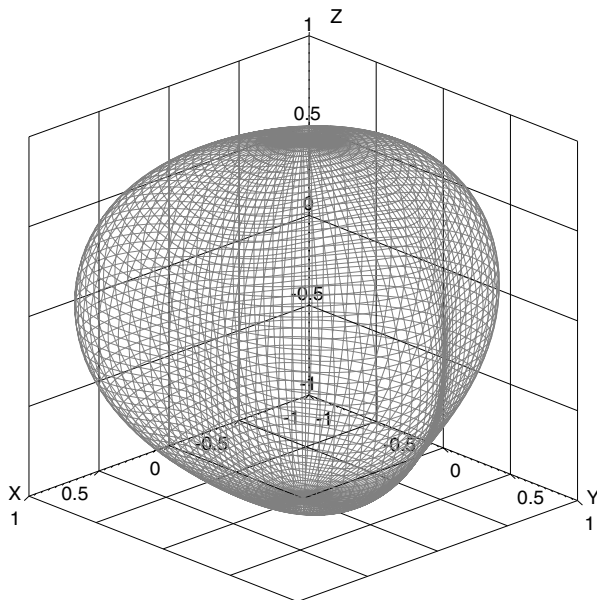


Fig. 2. The relation  $(\Gamma(\theta, \phi)/\Gamma_0)$  calculated for hcp crystal.

Table 1

Physical properties of Zn–Cd alloys used in the numerical calculations [15]

Liquidus slope	$m$	–2.8	K/wt.%Cd
Distribution coefficient	$k_0$	0.13	–
Diffusion coefficient	$D_L$	$3.0 \times 10^9$	$\text{m}^2/\text{s}$
Latent heat of fusion	$L$	444	J/kg K
Solid thermal conductivity	$\kappa_S$	96.0	W/mK
Liquid thermal conductivity	$\kappa_L$	164.0	W/mK
Gibbs–Thomson coefficient	$\Gamma$	$0.7 \times 10^7$	mK

in the  $\langle 10\bar{1}0 \rangle$  direction. Physical properties of Zn–Cd alloys used for calculations can be seen in Table 1.

The fundamental aim of this work was to evaluate some aspects of the microstructure details during the planar to cellular transition, using metallographic analysis of the samples and to extend the classical interpretation of morphological stability approach in order to interpret the experimental results.

## 2. Experimental

Directional growth of Zn–0.01wt.%Cd alloy was carried out in a Bridgman device. The alloy was prepared starting from Zn (99.999%) and Cd (99.99%) purity materials, carrying out the melting process in a SiC crucible which was painted internally with an alumina coating. Details of the experimental device, as well as the control of the solidification parameters  $G_L$  and  $V$ , and the procedure of quenching were described previously [16–18]. A schematic illustration of the device is shown in Fig. 3.

The set-up process of the surface for optical microstructural observation and analysis requires great attention, because this alloy can show plastic deformation during the mechanical polishing, giving rise to twinning formation and the re-precipitation of small grains on the surface. To

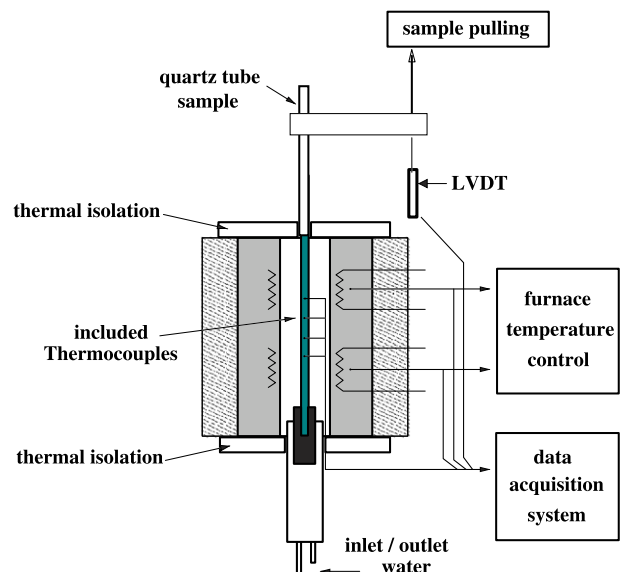


Fig. 3. Schematic view of the experimental device used in this work.

avoid these effects, an extremely slow and continuously refrigerated mechanical polishing was carried out. The microstructure was observed both in longitudinal views (in the direction of the growth) and in cross sections. The surface was set up by mechanical polishing with up to 600 grit SiC abrasive paper, refrigerated with water, and then with 6 and 3  $\mu\text{m}$  diamond powder refrigerated with alcohol. In some cases, chemical polishing with 200 g  $\text{Na}_2\text{CO}_3$ ; 6 ml  $\text{H}_2\text{SO}_4$  in 1000 ml distilled water solution was used for intervals of 1 min. Electrolytic polishing was also used, with 75 ml  $\text{H}_3\text{PO}_4$  in 625 ml ethanol solution, and 2–5 V direct current; bath temperature less than 10 °C. Electro-polishing also generates a thick surface oxide which allows us to detect changes in crystallographic orientation, when observed under polarized light.

### 3. Results

It is known that, from a qualitative point of view, for a given composition in an alloy with partition coefficient  $k_0 < 1$ , there exists a critical value for  $\left(\frac{G_L}{Vc_0}\right)$  which is given for the modified constitutional supercooling that predicts the limit for a planar–cellular transition. When this relation becomes smaller than the critical, a planar microstructure could evolve to a non planar one, where richer zones correspond to interface depressions. Even for very dilute alloys, the concentration in such depressions could achieve the eutectic composition [19,20].

Once the thermal gradient was fixed at  $G_L = 25 \text{ K/cm}$ , we start the experiments by the stabilization of growth

below the critical velocity, usually 50  $\mu\text{m/s}$  was chosen for this starting velocity. After a time growing at this velocity, not less than  $\tau = D_L/kV^2$ , the velocity was increased until it reached the desired value. Fig. 4a shows a photographic composition of the interface for a sample grown at  $V = 65 \mu\text{m/s}$ , where two grains can be easily identified. It may be noted that the critical wavelength of each grain is different. It is also apparent that planar morphology survives a longer time in the grain on the left, advancing a space of several microns in planar form over the other. Fig. 4b shows a similar situation for a sample obtained at  $V = 75 \mu\text{m/s}$ . In this case, the difference at the breakdown is not as evident as in Fig. 4a. Critical wavelengths and primary spacing after stabilization were determined from the micrographs.

### 4. Planar front stability

It is accepted that the growth morphology can be interpreted from the morphological stability theory as a first approximation [1,4–6]. Firstly, we assume that an initially planar front is growing in the  $z$  direction with velocity  $V$ . This planar front is perturbed by a spatio-temporal function

$$z = \delta \exp \left( \sigma t + i 2 \frac{\pi x}{\lambda} \right) \quad (3)$$

where  $\delta$  is the amplitude and  $\lambda$  is the wavelength of the perturbation. The growth rate  $\sigma$  can be determined by resolving the heat and diffusion equations for small  $\delta$ , where linear expansion of coefficients can be applied [5]. This

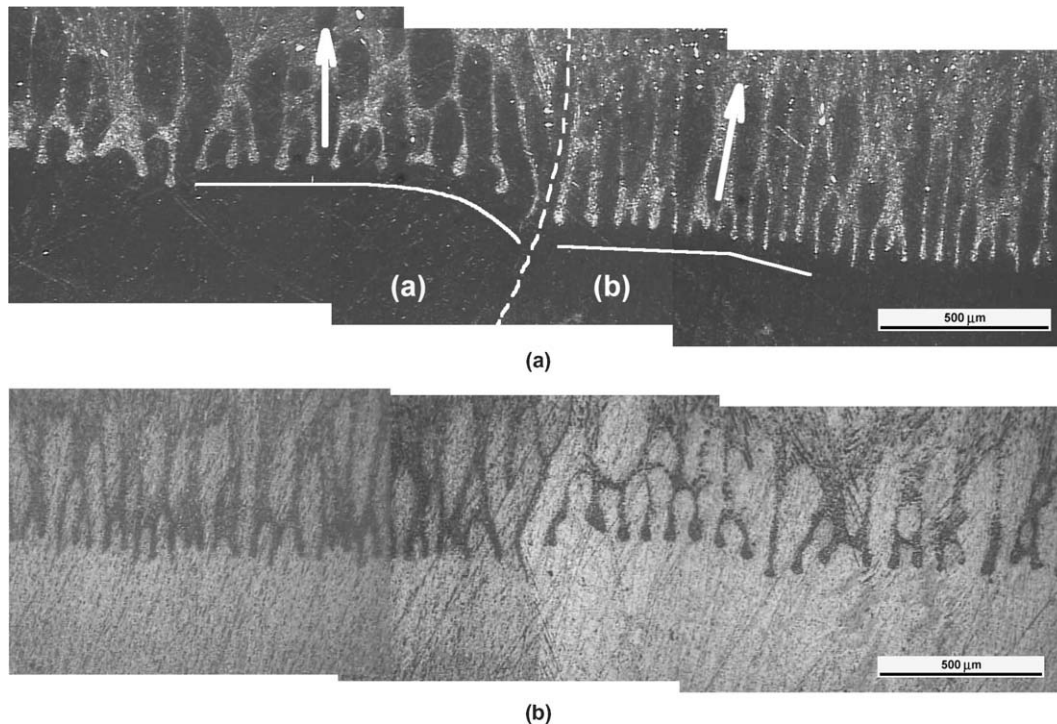


Fig. 4. Micrographs of planar front, Zn-0.01wt.%Cd,  $G_L = 25 \text{ K/cm}$ . Samples grown at (a)  $V = 65 \mu\text{m/s}$  and (b)  $V = 75 \mu\text{m/s}$ .

calculus results in a function that includes three terms with thermal, diffusive and capillary contributions:

$$\sigma = m_L G_C \xi_C - G - \frac{4\pi^2 T_m \Gamma(\theta, \phi)}{\lambda^2} \quad (4)$$

Negative values of  $\sigma$  for all  $\lambda$  determine a stable planar front, positive values determine an unstable condition and the particular case  $\sigma = 0$  defines the limit between stability and instability and then it is called marginal stability. In Eq. (4),  $T_m$  corresponds to the solvent equilibrium melting temperature,  $\Gamma(\theta, \phi)$  is the Gibbs coefficient, which now has an angular dependence in relation to  $\langle 0001 \rangle$  direction and  $G$  is a thermal gradient, averaged between solid and liquid with the thermal conductivities and it is given by

$$G = \left( \frac{\kappa_S G_S + \kappa_L G_L}{2\bar{\kappa}} \right) \quad (5)$$

where  $\bar{\kappa} = (\kappa_S + \kappa_L)/2$  is the average of solid and liquid thermal conductivities.  $G_C$  is the composition gradient, which for a planar front advancing at constant velocity can be written as

$$G_C = \left( \frac{VC_0(k_0 - 1)}{k_0 D_L} \right) \quad (6)$$

while  $\xi_C$  is a parameter which depends on  $V$  and  $\lambda$ , that remains close to unity, except at rapid solidification conditions [5], and can be written as

$$\xi_C = 1 + \frac{2k_0}{1 - 2k_0 - \sqrt{1 + \left(\frac{4\pi D_L}{V\lambda}\right)^2}} \quad (7)$$

For a given growth velocity between  $V_C$  (modified constitutional supercooling) and  $V_{abs}$  (absolute stability criterion),  $\sigma$  is positive for a wavelength interval [5,21], and shows a maximum  $\sigma_{max}$  for an intermediate wavelength  $\lambda_{\sigma_{max}}$ , as can be seen in Fig. 5a. The wavelengths  $\lambda_0^+$  and  $\lambda_0^-$ , for which  $\sigma = 0$ , define the marginal stability condition. Intermediate wavelengths between  $\lambda_0^-$  and  $\lambda_0^+$  have a propagation rate  $\sigma > 0$  and therefore,  $\lambda_0^\pm$  limit the instability–stability zone for the planar front. In Fig. 5b the values of  $\lambda$  for which  $\sigma = 0$  have been transported for the velocity range among  $V_C$  and  $V_{abs}$ . As explained after Fig. 2, the calculation was made for  $\Gamma_{\langle 10\bar{1}0 \rangle}$  and  $\Gamma_{\langle 0001 \rangle}$ , where we expect the maximum differences, and for the neutral  $\Gamma_0$ , for comparison.

## 5. Discussion

As was seen previously, Eq. (4) together with (5)–(7), allows us to interpret the planar front behavior. In these equations, the angular dependence of  $\Gamma(\theta, \phi)$  given by (2) was assumed. This approximation only takes into account that the misorientation of heat flux and primary growth direction only affects supercooling through surface energy and not from kinetic attachment.

Fig. 5 shows a stability map that was calculated for a composition of Zn–0.01wt.%Cd and a thermal gradient  $G_L = 25$  K/cm. In Fig. 5a the value of  $\sigma$  is calculated for an intermediate velocity. In this figure, the values of  $\lambda_0^+$  and  $\lambda_0^-$ , where  $\sigma = 0$ , and  $\lambda_{\sigma_{max}}$ , for which  $\sigma$  has a maximum value, are identified and transferred to obtain Fig. 5b. It can be noted that the correction is more evident for the smaller wavelengths and is practically negligible for the greater ones, because the capillary term depends on  $(\lambda^{-2})$ . The correction is also more appreciable at greater velocities. From both figures, it can be seen that the correction is a shift of critical values as  $\Gamma(\theta, \phi)$  becomes greater or less than  $\Gamma_0$ .

To interpret the experimental results, Fig. 6 shows the low velocity zone of the stability map, where measured determinations are added. In this figure, minimal  $\lambda_{min}$  and primary spacing  $\lambda_1$  are indicated. The primary spacing appears after an initial competition and a stabilization process. From the figure, it is evident that there are two well differentiated sets of experimental data points, characterized by the different crystalline orientations of each grain.

In both cases,  $\lambda_1$  is close to the  $\lambda_{\sigma_{max}}$  line, for both data sets. At the same time,  $\lambda_{min}$  remains close to the  $\lambda_0^-$  value, which is in agreement with observations made in Al–Cu

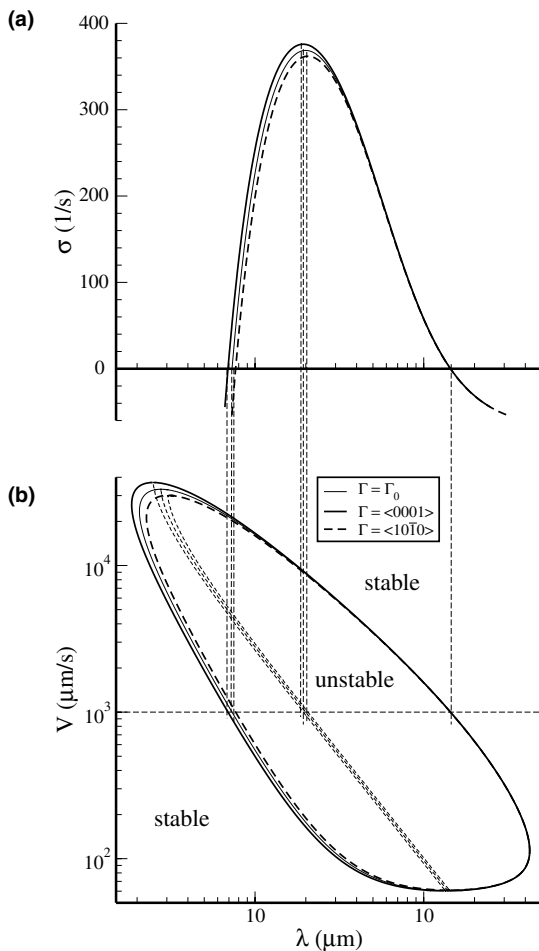


Fig. 5. Stability map generated for Zn–0.01%Cd and a thermal gradient  $G_L = 25$  K/cm.



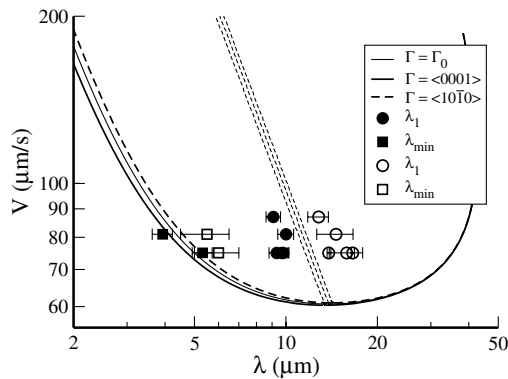


Fig. 6. Experimental data for  $\lambda_1$  and  $\lambda_{\min}$  obtained during the destabilization of planar front in different grains. The values are located in the stability map of Fig. 5, in the low velocity zone.

dilute alloys [6,7]. Also for both sets, the relation between  $\lambda_1/\lambda_{\min}$  remains close to the  $\lambda_{\sigma_{\max}}/\lambda_0^-$  that can be obtained from the stability map for this system.

From the analysis of the experimental data and the stability map in Figs. 5 and 6, it can be seen that the incorporation of the angular dependence in surface tension shows the same deviation as was experimentally observed, but it is not enough to explain the full deviation. This difference could have a relation not only to the surface energy anisotropy, but also to a kinetic attachment effect. Another possibility could be that the value used for  $\epsilon_0$ , which is typical value for fcc crystals, would be underestimated for our hcp system and must be evaluated or measured for this geometry by other means. Following Eq. (2), a greater value of  $\epsilon_0$  will not change the geometry in Fig. 2, but will show a more important deformation of the spherical surface given by  $\Gamma_0$  and thus a greater difference in Figs. 5 and 6.

## 6. Conclusions

In this work, the stability of a planar solidification front of an hcp: Zn–0.01%Cd alloy directionally grown was studied. Experimentally, we found that the critical values  $\lambda_c$  and  $V_c$  depend on the growth direction. For this reason, the scope of the morphological stability theory was analyzed. In the original form, this theory does not incorporate the anisotropy contribution of the surface energy. By adding an angular dependence in this term, this model shows a shift for primary spacing (actually the  $\lambda_{\sigma_{\max}}$  value), as for  $\lambda_0^-$ , which corresponds to the minimal wavelength that could appear during the destabilization.

Although the application of the model shows a minor displacement for the critical velocity  $V_c$ , the difference is

more notable for higher growth velocities, and also, as it can be expected for the smaller wavelengths, due to the square inverse dependence on  $\lambda$  of the capillary term.

Finally, the experimentally observed difference is greater than that expected for this correction. The reason could be that the value of the  $\epsilon_0$  for this system may be underestimated. As a first step, we used the typical value for metallic systems, but it would be interesting to calculate or measure it by other means.

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